Phd Project

KESS 2 Project - Reduced Order Modelling in Nuclear

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An introduction.

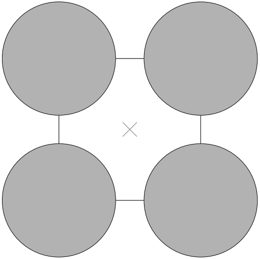
- I have a background in mathematics starting out with a four year theoretical physics degree in Aberystwyth.
- I am working on this project at Bangor with the support of Answers, part of the Wood Group and AMCG at Imperial Collage.
- We aim to apply reduced order modeling to sub-channel models.

Why use a reduced order model?

- Numerical simulations are a key part of understanding many physical systems various fields such as climatology, economics, biology and engineering.
- Simulations often involve a large number of partial differential equations (PDEs).
- Typical discretization techniques yield a system of n ordinary differential equations (ODEs).
- This can make the number of degrees of freedom in the order of 10^6 or more.
- Whence a complicated enough problem will be prohibitively expensive to compute - both in storage and CPU cost.

A Sub-channel model.

- The sub-channel model gives quick results on local thermal-hydraulic information which could be used to determine reactor core power distribution, critical heat flux etc.
- Results of the model are in some sense an average over each control due to lacking any further spacial resolution.
- Many properties in the model, like heat transfer from a rod to the fluid are empirically determined and used as inputs for the model, partially with the introduction of multiphase models.

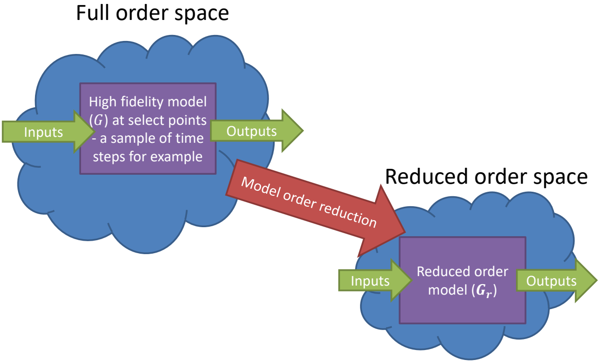


Control Volume centred on sub-channel.

In this KESS project we hope to ”speed up” sub-channel model simulations.

- Reactor technology makes use of vital system codes which model an entire plant.
- Also CDF is used to model the behavior of a particular element of the plant.
- ROM is a relativity new technique that can produce a speed up of orders of magnitude.

Reduced order modelling (ROM).



Reduced order modelling constructs an approximation of the original model with acceptable accuracy in less time.

The snapshot matrix.

- To begin constructing a reduced order model we must collect the data from our high fidelity model.
- A matrix is formed with the column vectors being solutions - we sample them for time steps for example

$$U_{Snapshots} = \begin{pmatrix} u(x_1; t_1) & \dots & u(x_1; T_{end}) \\ \vdots & \ddots & \vdots \\ u(x_n : t_1) & \dots & u(x_n; T_{end}) \\ . & & \end{pmatrix}. \quad (1)$$

- We modify the snapshot matrix as follows

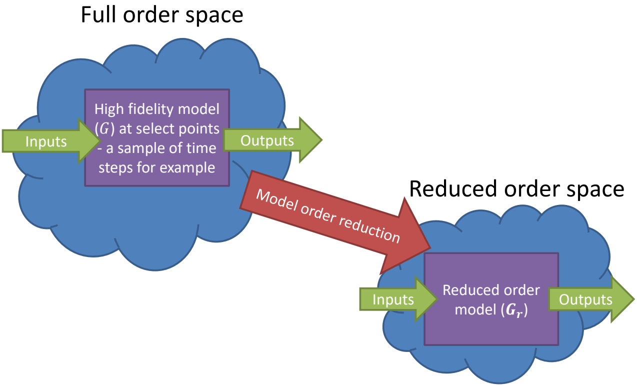
$$U_{Modified} = U_{Snapshots} - \bar{U}_{Averaged} \quad (2)$$

where

$$\bar{U}_{Averaged} = \frac{1}{S} \sum_{s=1}^{S} U_{Snapshots} \quad (3)$$

and S is the number of snapshot vectors.

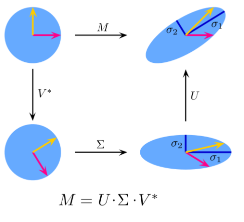
Proper orthogonal decomposition (POD) gives a set basis functions for the reduced order space.



The basis functions can be combined to get the reduced order model.

Obtaining POD basis functions.

- These basis functions are optimal and are obtained by taking the singular value decomposition (SVD) of the modified snapshot matrix $U_{Modified}$.



Singular value decomposition of a matrix M separates the rotation and reflection from stretching.

- The SVD gives a decomposition $U_{Modified} = U\Sigma V^*$ with unitary matrices U, V^* and Σ a diagonal matrix with monotonically decreasing entries.

The POD basis functions are the column vectors of the matrix U - we label them Φj with $j = 1$ corresponding to the largest singular value, $j = 2$ the next largest etc...

- The first few POD basis functions capture most of the information needed to give our approximation of the full order model.
- We can take $P << n$ basis functions to get a sufficiently accurate reduced order model.
- Solution variables can be represented through

$$y = \bar{y} + \sum_{j=1}^{P} \alpha_j \Phi_j, \tag{4}$$

where α_j are coefficients arising from a ROM technique and $\bar{y}$ is the column wise average of a snapshot matrix.

The coefficients can either be obtain by projection onto the original dynamics (intrusive) or through interpolation using radial basis functions (non-intrusive).

- The first technique, Galerkin Projection will give $P << n$ ODEs and we can solve it to get the coefficients α_j as a function of x. If we think of are original PDE as n ODEs with linear and nonlinear parts L and N

$$\frac{d\mathbf{u}(t)}{dt} = L\mathbf{u}(t) + N(\mathbf{u}(t)) \tag{5}$$

then

$$\frac{d\mathbf{a}(t)}{dt} = \mathbf{\Phi_P^T} L \mathbf{\Phi_P a}(t) + \mathbf{\Phi_P^T} N(\mathbf{\Phi_P a}(t)). \tag{6}$$

gives the evolution of the coefficients needed to reconstruct a rank P approximation.

The non-intrusive ROM can be done without reference to the original dynamics.

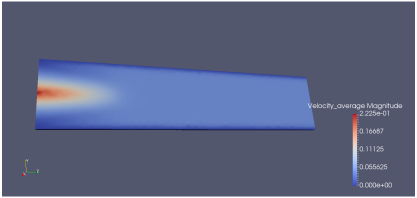
- Radial Basis Functions gives a mapping $\hat{f}(\alpha_j^{n-1}) = \alpha_j^n$ where α^n is a POD basis coefficient at time n.
- The form of $\hat{f}$ is rather like a simple single layer neural network

$$\hat{f}_i(\alpha^{n-1}) = \sum_{j=1}^{N} w_i^{n-1}(j)\phi(\|\alpha^{n-1} - \alpha_j\|), \quad i \in \{1, 2, \ldots, P\}. \tag{7}$$

- The weights w_i are given by the data in the snapshot matrix and are associated with centre points α_j.
- ϕ can be any real valued function, typically a Gaussian or Polyharmonic spline.

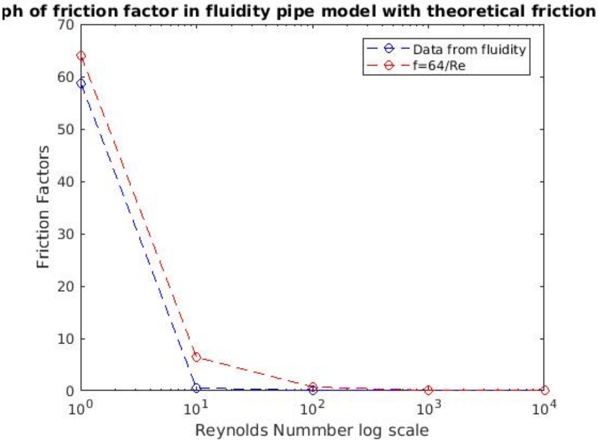
Aims for Phd.

- I aim to combine these two approaches by integrating the radial basis functions into a solver. My hope is this will give a solver that can be conservative, resolve at a sub-grid scale and capture nonlinear multiphase flow.
- This will be done by separating the approximation into a coarse scale component and a contribution from the sub grid scale.
- I aim to recover friction factors at different Reynolds numbers from the results as validation.

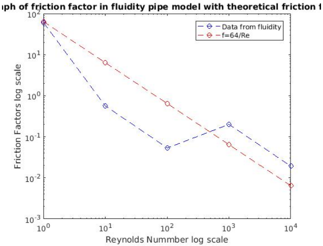


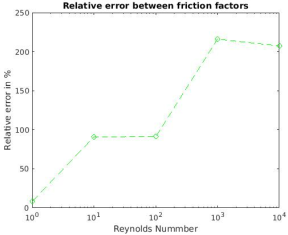
Flow in pipe.

Before using HPC data showed right trend but was bad.



Clear in log-log plot.





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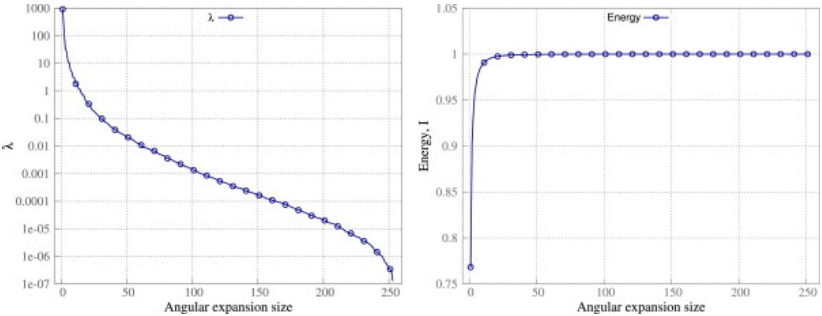
Proper orthogonal decomposition (POD) has shown to be powerful in CDF and neutronics application.

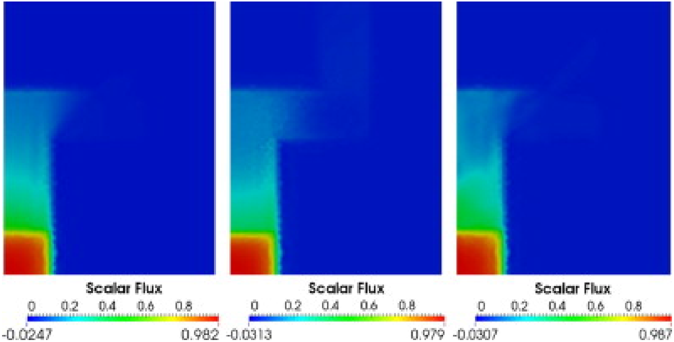
- For example in the AMCG at Imperial Collage, **Buchan, AG, Calloo, AA, Goffin, MG, Dargaville, S, Fang, F, Pain, CC, Navon,IM. 2015.**
  - A POD reduced order model for resolving angular direction in neutron/photon transport problems. Journal of Computational Physics, 296, 138–157.
  - An example in the paper is radiation transport through a dog leg shaped duct.

The first six POD basis functions.



Here the plots show the reduction of the singular values and how much information(energy) they capture. One basis function alone captures above 75% of the information whilst 21 basis functions exceed 99.7%.

Scalar flux solution.



High resolution solution, lower resolution solution and approximation with 10 POD basis functions.

Thank you